

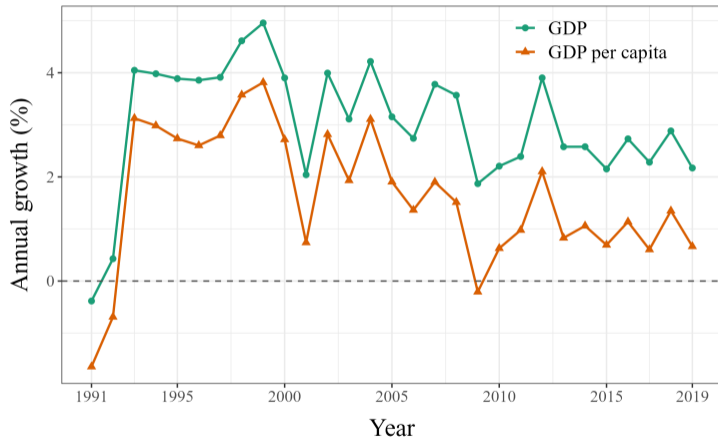
# Uneven Growth, Redistribution and Inequality: The Australian Case

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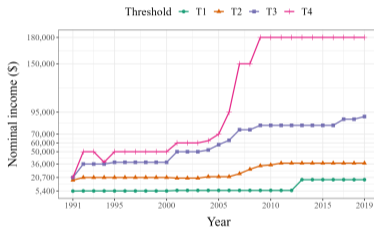
TTPI Seminar  
16 June 2023

## Australia: Three decades of uninterrupted growth

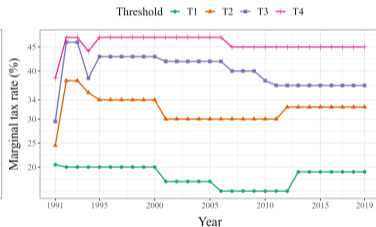


# Australia: A progressive income tax system

with a series of major tax reforms in the 2000s.



(a) Tax thresholds



(b) Statutory marginal tax rates

# Questions

1. How evenly/unevenly was economic growth distributed?
2. To what extent could a progressive tax and transfer system moderate uneven gains and reduce inequality?

# Our paper

## Part I: Empirical analysis

- Data: ALife 1991-2019 (~1 million individuals per year, longitudinal).
- Two approaches to measuring income growth and inequality:
  - Point-in-time statistics (29 years).
  - Lifetime statistics (9 cohorts)

## Part II: Structural analysis

- Dynamic general equilibrium lifecycle model for Australia.
- Counterfactual analysis of alternative tax and transfer policies
  - Exploring the possibilities and costs of redistributing via taxes and transfers

# Three perspectives on growth and inequality

## Part I: Empirical analysis

- Data: ALife 1991-2019 (~1 million individuals per year, longitudinal).
- Two approaches to measuring income growth and inequality:
  - **Point-in-time** statistics (29 years).
  - **Lifetime** statistics (9 cohorts)

## Part II: Structural analysis

- **Long run** Dynamic general equilibrium lifecycle model for Australia.
- Counterfactual analysis of alternative tax and transfer policies
  - Exploring the possibilities and costs of redistributing via taxes and transfers

## Main results

1. The benefits of economic growth were distributed unevenly.
  - disproportional gains at the top, bottom groups left behind
2. Progressive taxes and transfers played an important role in moderating uneven gains.
  - but not sufficient to curb rising inequality
3. Lifetime income inequality is much lower.
  - the potential biases of the point-in-time (cross-sectional) approach
4. Trade-offs between equity and efficiency in dynamic general equilibrium
  - the limits to redistribution via higher tax-transfer progressivity

## Related literature

### Income dynamics and inequality in advanced economies

Piketty and Saez 2003; Krueger et al. 2010; Guvenen et al.; Saez and Zucman 2020; Heathcote, Storesletten and Violante 2020; Lippi and Perri 2023; Guvenen et al. 2021; De Nardi et al. 2021; Guvenen et al. 2023.

### Inequality in Australia

Leigh 2005; Wilkins 2015; Chatterjee, Singh and Stone 2016; Kaplan, Cava and Stone 2018; Productivity Commission 2018; Fisher-Post, Herault and Wilkins 2022; Herault and Azpitarte 2015; Tran and Zakariyya 2021; Tin and Tran 2023.



# Part I: Empirical Analysis

# Measurement and data

## Income concepts

$$c_{j,t}^i + a_{j+1,t}^i = \underbrace{\underbrace{w_{j,t}^i n_{j,t}^i + r_{j,t}^i a_{j,t-1}^i}_{y_{j,t}^{i,market}: \text{market income}} - \underbrace{t_{j,t}^i}_{\text{tax}} + \underbrace{tr_{j,t}^i}_{\text{gov. transfer}}}_{y_{j,t}^{i,post-gov.}: \text{post-government income}} + \underbrace{b_{j,t}^i}_{\text{pri. transfer}} + \underbrace{a_{j,t}^i}_{\text{asset}}, \quad (1)$$

- Point-in-time variables:  $y_{j,t}^{i,market}$ ,  $t_{j,t}^i$ ,  $tr_{j,t}^i$  and  $y_{j,t}^{i,post-gov.}$ .
- Lifetime variables:  $LY_{t_{\kappa}}^{i,market} = \sum_{j=j_1}^J y_j^{i,market}$ , and  $LY_{t_{\kappa}}^{i,post-gov.} = \sum_{j=j_1}^J y_j^{i,post-gov.}$

## Sample restrictions

- 20 years and above.
- Non-negative market income, tax and transfers.

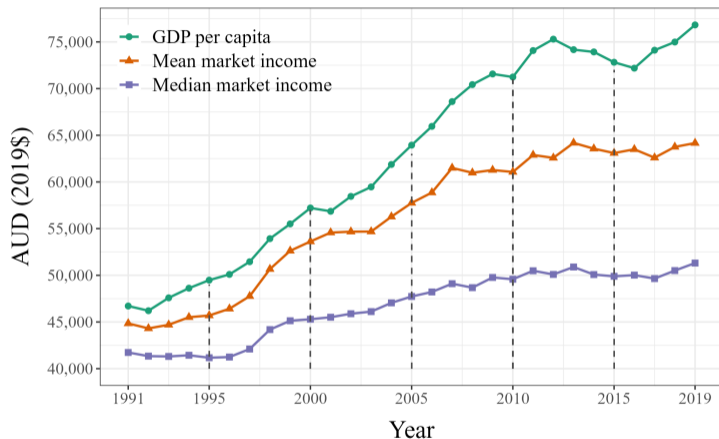
## Data sample

Table: Frequency of individuals - ALife data and sample

Year	Data	Sample	% Included
1991	983,476	736,584	75
1995	1,012,619	770,549	76
2000	1,076,254	838,057	78
2005	1,203,103	897,518	75
2010	1,338,919	976,803	73
2019	1,530,918	1,185,275	77

- All income and tax variables in 2019\$ AUD.

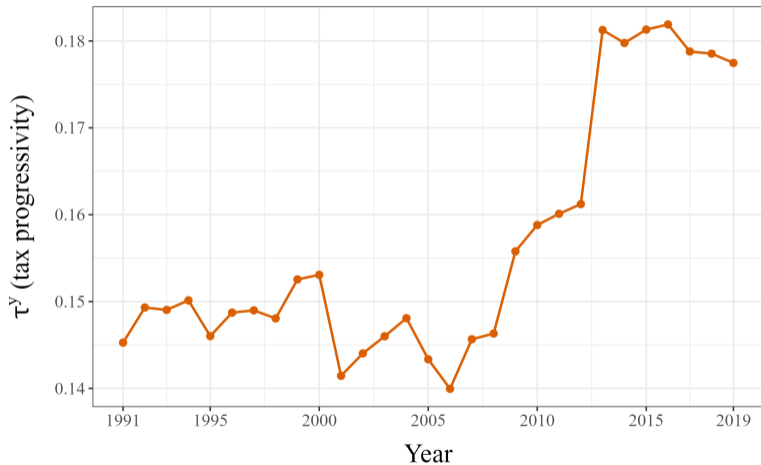
# Market income growth



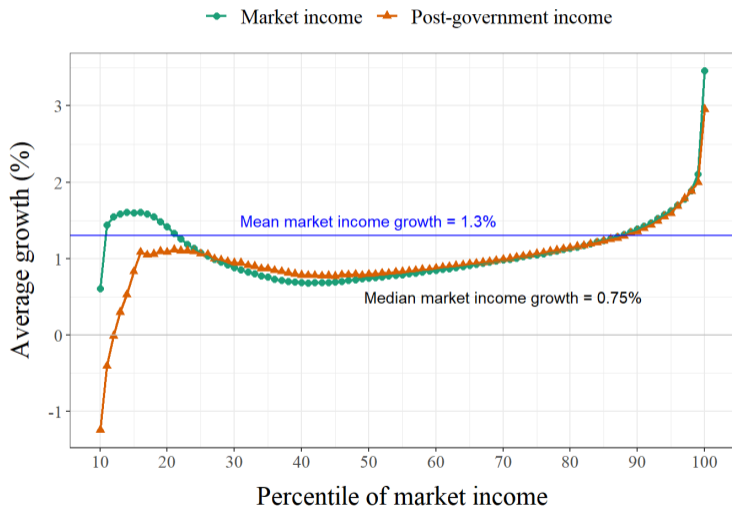
Growth (%)	1991 - 2019	1991 - 1995	1995 - 2000	2000 - 2005	2005 - 2010	2010 - 2015	2015 - 2019
Mean market income	1.30	0.48	3.26	1.50	1.14	0.67	0.43
Median market income	0.75	-0.34	1.94	1.06	0.77	0.14	0.71
GDP per capita	1.81	1.46	2.95	2.26	2.20	0.46	1.36

# Rising tax progressivity

Income tax has become more progressive



# How evenly was growth distributed, and re-distributed?

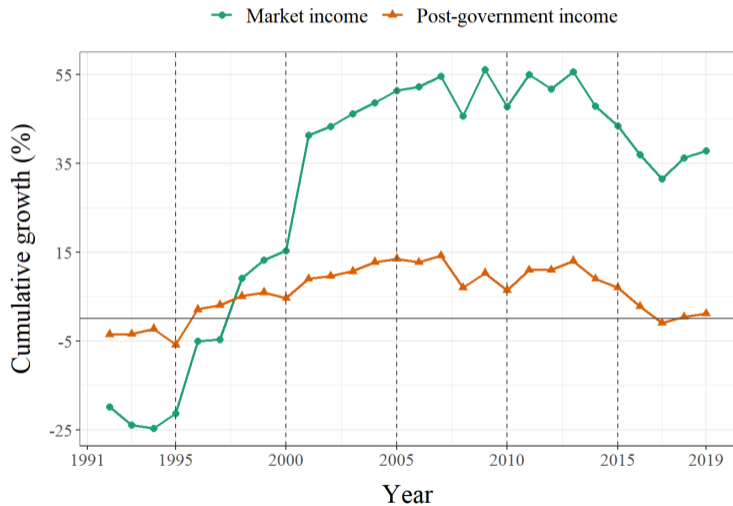


# Uneven growth across the distribution over time



# Cumulative growth: Bottom 20%

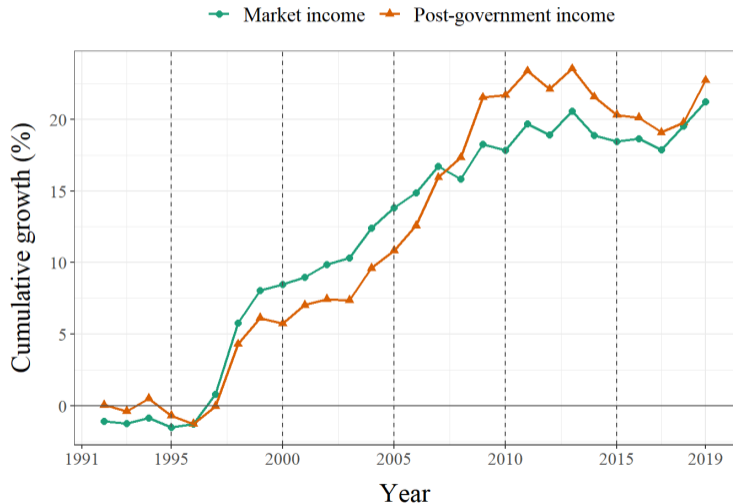
Significant market gains, but very little growth in post-government income.



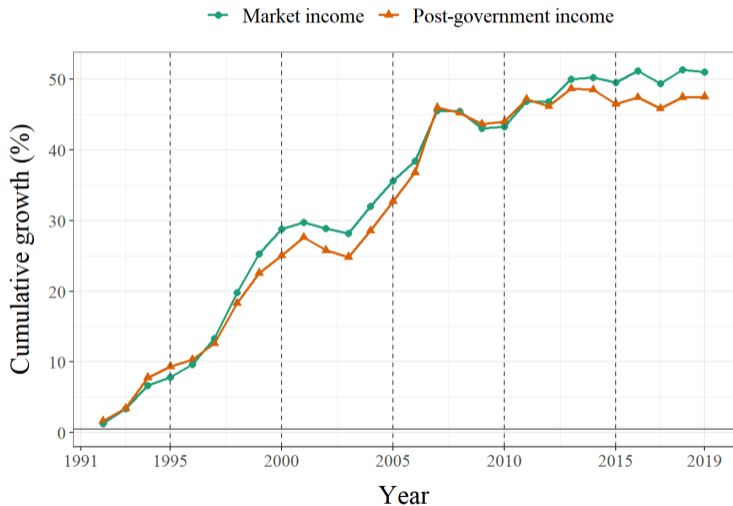


# Cumulative growth: Middle 40-60%

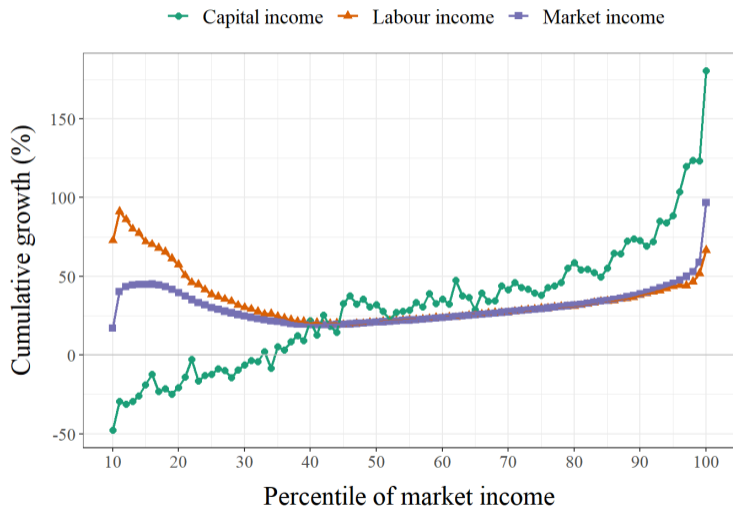
Middle incomes gained from tax changes since 2007



## Cumulative growth: Top 20%

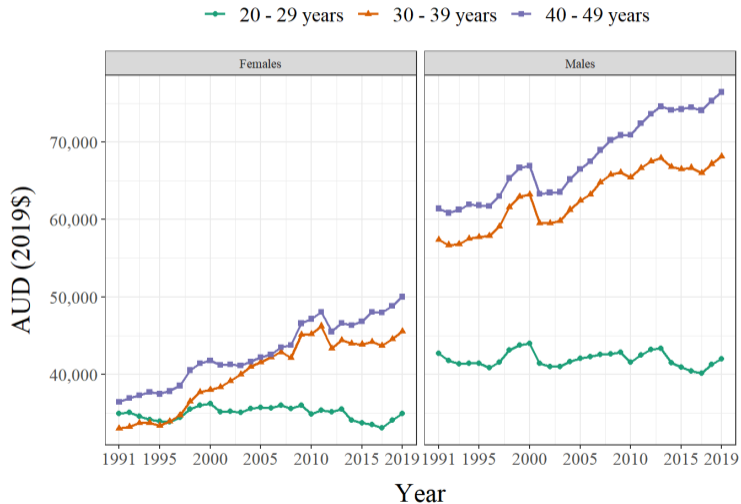


# Uneven growth: Role of capital and labor



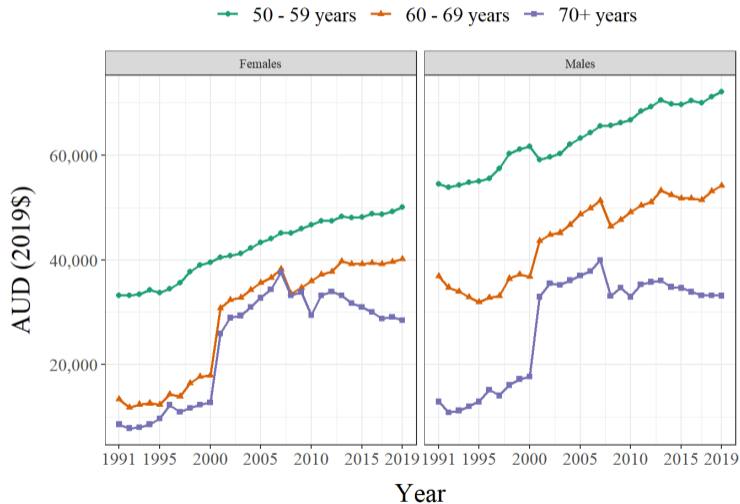
# Uneven growth: Age cohort and gender

Median market income by age group, year and sex (20 - 49 years)



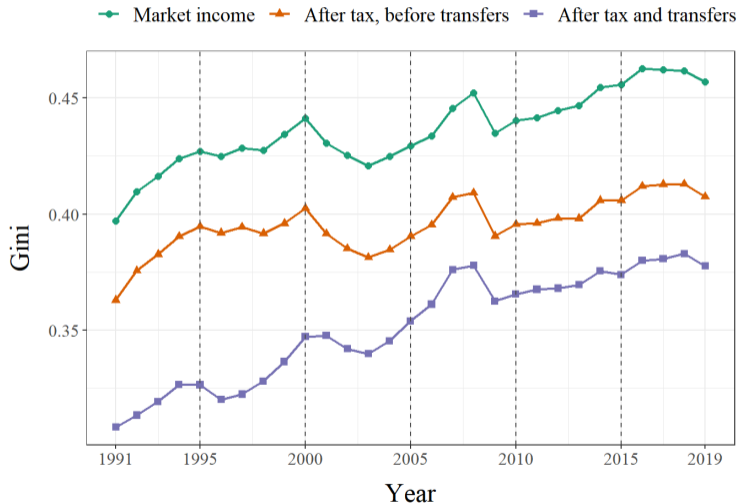
# Uneven growth: Age cohort and gender

Median market income by age group, year and sex (50 years and over)



# Uneven growth → Rising income inequality

Trends in the Gini coefficient



# A closer examination of redistribution

## Measuring the redistributive effect of taxes and transfers

- Reynolds and Smolensky (1977)

$$RE = Gini_{pre} - Gini_{post} \quad (2)$$

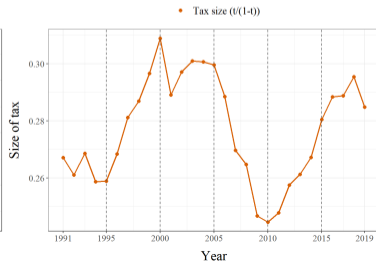
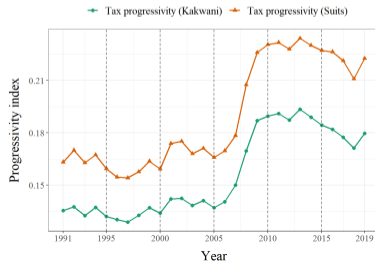
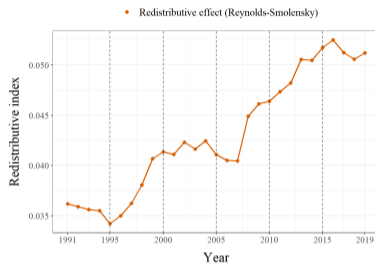
- Decomposition (Lambert, 2001)

[1] Size: Average rate of tax on **net** income

$$RE = \underbrace{\frac{t}{1-t}} \times \underbrace{K_T} \quad (3)$$

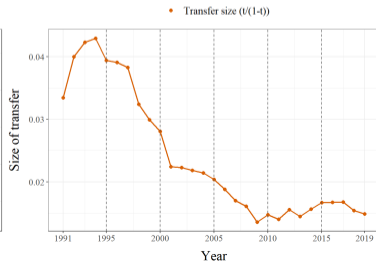
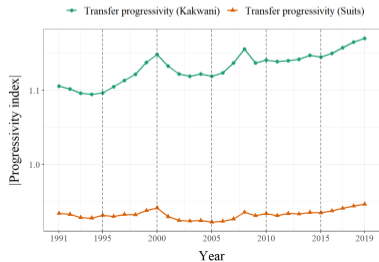
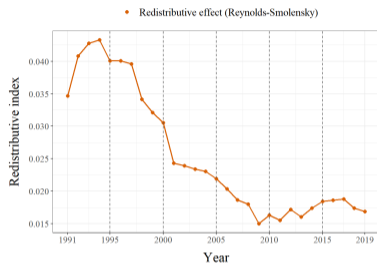
[2] Progressivity: Kakwani index

# Progressive income tax played a large role





# Transfers have been highly progressive, but size decreased

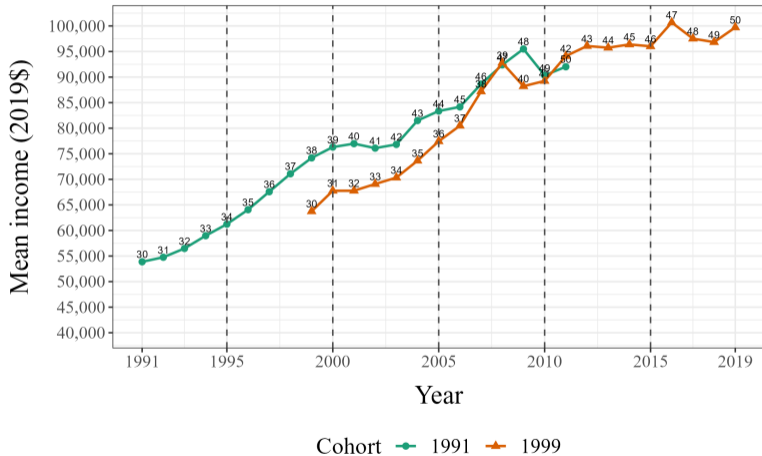


## From point-in-time to lifetime approach

- Point-in-time data pool all individuals at different stages of their lifecycles.
  - Annual incomes can be transitory.
  - Extensive margin of labour and long run mobility
- Point-in-time statistics are potentially biased.
- A more complete picture: Statistics based on lifetime resources

# Lifecycle profile

Mean market income by age for two cohorts (30-50 years)



## Lifetime approach

Sum of annual incomes.

$$LY_{t_{\kappa}}^{i,market} = \sum_{j=j_1}^J w_{j,t+j-1}^i n_{j,t+j-1}^i + \sum_{j=j_1}^J r_{j,t+j-1}^i a_{j,t+j-1}^i \quad (4)$$

- Group individuals by cohort and index each cohort by the year they entered the sample  $t_{\kappa}$ .
- Track each cohort for 20 years from the year they turned 30 ( $j_1 = 30$ ) till the year they turned 50 ( $J = 50$ ).

## We track 9 cohorts

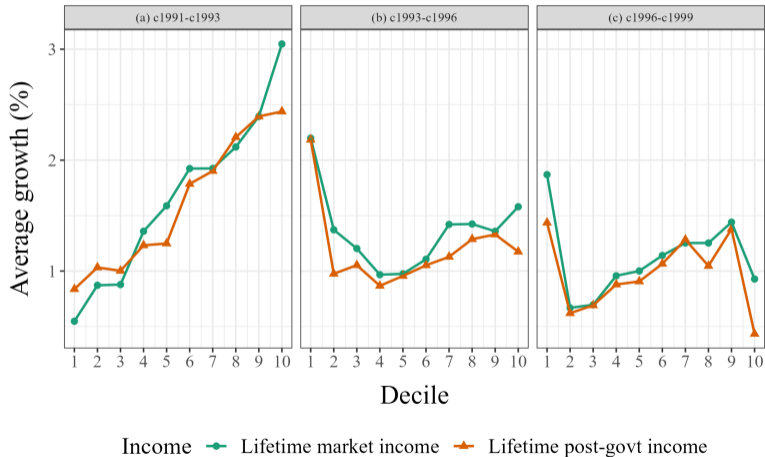
From the year they turn 30 to the year they turn 50. (c1991 turned 30 in 1991...)

Table: Sample composition by cohort and gender

	Cohort	Birth year	Last year	N	Males (%)	Females (%)
Older	c1991	1961	2011	12,447	60	40
	c1992	1962	2012	12,454	61	39
	c1993	1963	2013	12,453	60	40
Middle	c1994	1964	2014	12,311	60	40
	c1995	1965	2015	11,834	60	40
	c1996	1966	2016	11,711	59	41
Younger	c1997	1967	2017	11,754	58	42
	c1998	1968	2018	11,779	57	43
	c1999	1969	2019	12,501	57	43

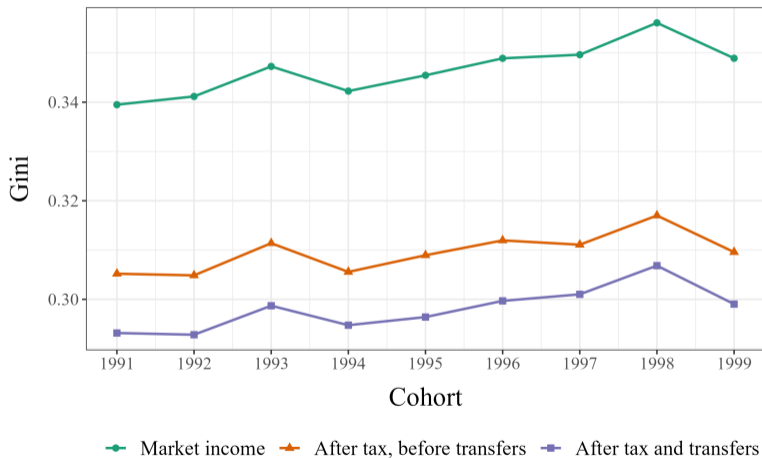
# Lifetime income growth

Growth **between** cohorts by deciles of lifetime market income (growth rates averaged within each of the 3 groups of cohorts).

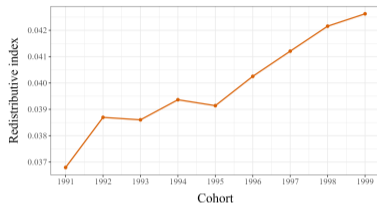


# Lifetime inequality

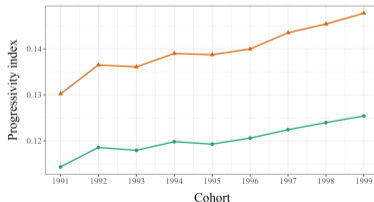
Inequality **within** cohorts is fairly stable



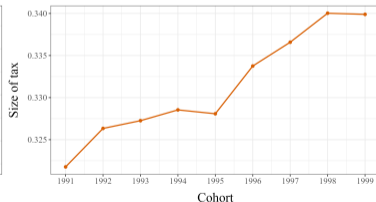
# Redistributive effect of lifetime tax



• Redistributive effect (Reynolds-Smolensky)



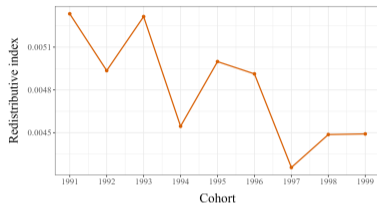
• Tax progressivity (Kakwani) • Tax progressivity (Suits)



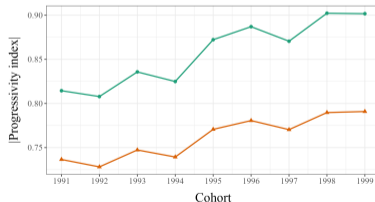
• Tax size  $t/(1-t)$



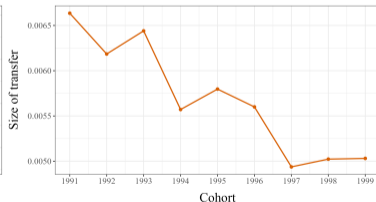
# Redistributive effect of lifetime transfers



• Redistributive effect (Reynolds-Smolensky)



• Tax progressivity (Kakwani) • Tax progressivity (Suits)



• Tax size (t/(1-t))

# Caveats

Lifetime approach provides a good overview of inequality, but.....

- “Lifetime”: 30 - 50 years.
- More detailed public transfers.
- Interactions between market income, tax, transfers and incentives.
- **Solution:** Structural lifecycle model.

# Part II: Structural Analysis

# Model

## SOLGA - Stochastic General Overlapping Generations Model for Australia

- Large scale computable **general equilibrium** OLG model (Auerbach and Kotlikoff, 1987)
- Heterogenous households who face **uninsurable labour productivity risk**. (Bewley, 1986; Huggett, 1993; Aiyagari, 1994)
- Government (**Australian tax-and-transfer system**)
- Age  $j = 20$  to  $j = 89$ , **life-cycle** and **survival probability risk**.

# Labour productivity

## Innate skill types

$$q \in \{low, mid, high\}$$

## Labour productivity

Quintiles by age (hump-shaped)

$$\eta_{z,j} \in \{\eta_{1,j}, \eta_{2,j}, \eta_{3,j}, \eta_{4,j}, \eta_{5,j}\}$$

$$\pi_{z,j}^q (\eta_{z,j+1} | \eta_{z,j})$$

Transition probability matrix (differs by skill type)

## Household choices

$$a_{j+1} = ra_j + \eta_{z,j} (1 - l_j) w + p_{j \geq J^p} + s t_{j < J^p} - t(y_j) - (1 + \tau^c) c_j + a_j$$
$$a_j \geq 0, 0 < l_j \leq 1$$

## Household incomes

$$a_{j+1} = \overbrace{ra_j + \eta_{z,j} (1 - l_j) w}^{y_j(\text{taxable income})} + p_{j \geq JP} + st_{j < JP} - t(y_j) - (1 + \tau^c) c_j + a_j$$

$y_j^m(\text{market income})$

$$a_j \geq 0, 0 < l_j \leq 1$$

## Transfers to households

$$a_{j+1} = y_j^m + p_{j \geq J^p} + st_{j < J^p} - t(y_j) - (1 + \tau^c) c_j + a_j$$

- Public transfers before 65 years (progressive)

$$st_{j < J^p} = st(j, \eta_{z,j})$$

- Pension 65 and above

$$p = \begin{cases} p^{\max} & \text{if } y^m \leq \bar{y}_1 \\ p^{\max} - \omega^y (y^m - \bar{y}_1) & \text{if } \bar{y}_1 < y^m < \bar{y}_2 \\ 0 & \text{if } y^m \geq \bar{y}_2 \end{cases}$$



## Taxes on households

$$a_{j+1} = y_j^m + p_{j \geq J^p} + s_{t_{j < J^p}} - t(y_j) - (1 + \tau^c) c_j + a_j$$

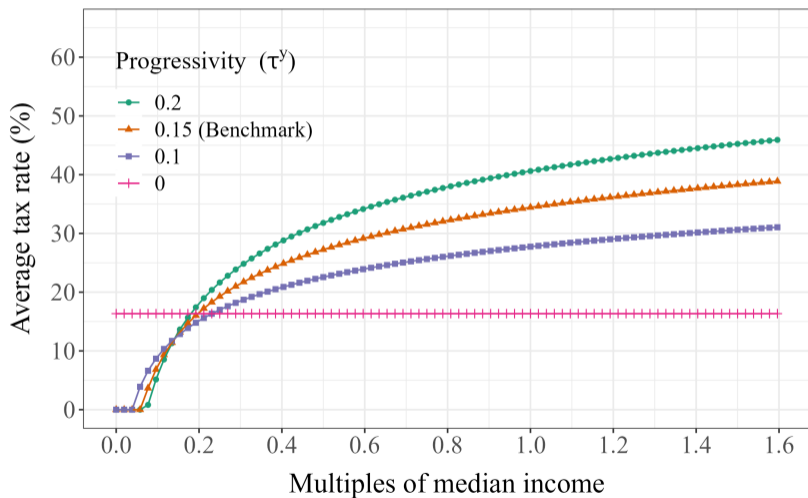
- Income tax

$$t(y_j) = \max \left[ 0, y_j - \lambda y_j^{1-\tau^y} \right]$$

- Consumption tax

# Tax function

$\downarrow \tau^y \implies$  less progressive,  $\downarrow \left(\lambda^{\frac{1}{\tau^y}}\right)$  tax-free threshold



## Household problem

$$V^j(\chi_j) = \max_{c_j, l_j, a_{j+1}} \left\{ u(c_j, l_j) + \beta \psi_{j+1} \sum_{\eta_{z,j+1}} \pi_{z,j}^{\theta}(\eta_{z,j+1} | \eta_{z,j}) V^{j+1}(\chi_{j+1}) \right\} \quad (5)$$

subject to:

$$a_{j+1} = \underbrace{ra_j + \eta_{z,j} (1 - l_j) w}_{y_j^m \text{ (market income)}} + p_{j \geq J^p} + st_{j < J^p} - t(y_j) - (1 + \tau^c) c_j + a_j$$

$$a_j \geq 0, 0 < l_j \leq 1$$

## Government

$$Tax = \overbrace{\sum_j t(y_j) \mu(\chi_j)}^{\text{Income tax}} + \overbrace{\sum_j t(c_j) \mu(\chi_j)}^{\text{Consumption tax}} + \overbrace{\tau^f (AK^\alpha H^{1-\alpha} - wH)}^{\text{Company income tax}} \quad (6)$$

$$Expenses = \overbrace{\sum_j p_j (y_j^m) \mu(\chi_j)}^{\text{Age-pension}} + \overbrace{\sum_j st_j (\eta_{j,j}) \mu(\chi_j)}^{\text{Other public transfers}} + \overbrace{G + rD}^{\text{Other expenses}} \quad (7)$$

## Benchmark economy

	Parameters	Measure	Model performance	
			Data	Target
Labour income	Labour productivity.	Gini	0.5	0.5
Taxable income	Labour productivity.	Gini	0.4	0.4
Income tax	$\lambda = 0.6557$ $\tau^y = 0.15$ (estimated)	Share of GDP (%)	16	11
		Suits index	0.17	0.19
		Kakwani index	0.14	0.17
		Tax size	0.3	0.3
		Redistributive effect	0.04	0.04
Public transfers	Estimated by wage quintile.	Share of GDP (%)	8	8
Pension	$p^{\max} = 0.06$ , $\omega^y = 0.5$ $y_1 = 0.0126$	Share of GDP (%)	2	2
		Pension participation rates by skill and age.		
Post-govt income	Matching this distribution is a combination of all the other income components.	Gini	0.34	0.34

Data sources: World Development Indicators (WDI) database, ALife, HILDA, OECD-SOCX: Social expenditure database of the OECD.

## Other parameters

Parameter	Value	Source/Target
Population growth rate	$n = 1.3\%$	WDI
GDP per capita growth rate	$g = 2.24\%$	WDI
Interest rates	$r = r^w = 1.04\%$	Investment share of GDP
Inter-temporal elasticity of consumption	$\sigma = 2$	
Share parameter for leisure	$\gamma = 0.3$	Labour supply over the life cycle
Discount factor	$\beta = 0.97$	Household savings share of GDP

Data: WDI: World Development Indicators, ABS: Australian Bureau of Statistics.

# Experiments

## 1. To what extent would more progressive income tax reduce inequality?

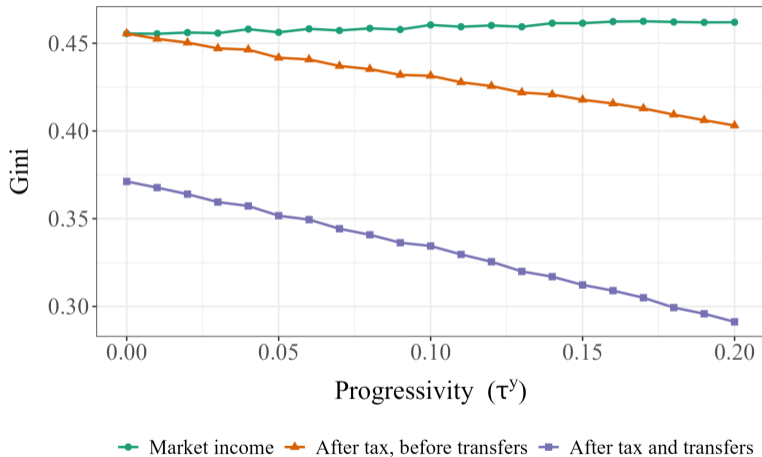
- $\uparrow \tau^y$  while (others including public transfer system at benchmark)
- What happens to inequality?
- What are the trade-offs?

## 2. To what extent would more generous public transfers reduce inequality?

- Change the level of all transfers from benchmark (150%, 50% and 0%).
- What happens to inequality?
- What are the trade-offs?

# 1. Changing tax progressivity

More progressive income tax can reduce income inequality





# 1. Changing tax progressivity

Comes at the cost of lower work hours, saving and output.

	$\tau^y = 0.15$ (Bench.)	$\tau^y = 0.2$ (Higher)	$\tau^y = 0.1$ (Lower)	$\tau^y = 0$ (Flat tax)
Hours ( $\% \Delta^{Bench}$ )				
- Aggregate	0.0	-5.44	6.67	18.2
- Low skilled	0.0	-6.11	8.2	20.85
- Medium skilled	0.0	-5.56	6.35	18.22
- High skilled	0.0	-4.97	6.42	16.95
Savings ( $\% \Delta^{Bench}$ )				
- Aggregate	0.0	-17.95	25.89	83.71
- Low skilled	0.0	-16.86	21.08	67.25
- Medium skilled	0.0	-17.85	27.18	80.87
- High skilled	0.0	-18.87	27.11	99.48
Output ( $\% \Delta^{Bench}$ )	0.0	-5.16	6.51	17.61

## 2. Changing transfer generosity

Increasing transfer generosity significantly reduces income inequality

	Bench.	150% $\Delta^{bench}$	50% $\Delta^{bench}$	0% $\Delta^{bench}$
<u>Income inequality (Gini)</u>				
Labour income	0.52	0.54	0.47	0.45
Capital income	0.63	0.66	0.55	0.44
Market income	0.46	0.45	0.44	0.41
After tax income	0.42	0.41	0.40	0.37
Net income	0.31	0.26	0.35	0.37
<u>Redistributive effect</u>				
Tax	0.04	0.05	0.04	0.04
Net	0.11	0.13	0.06	0.04

## 2. Changing transfer generosity

But it comes at the cost of lower work, savings, output and higher market income inequality.

	Bench.	150%	50%	0%
<hr/>				
<u>Hours worked (<math>\% \Delta^{bench}</math>)</u>				
- Aggregate		-8.08	16.08	29.63
- Low		-10.41	21.09	38.67
- Medium		-8.90	17.86	32.99
- High		-5.85	11.25	20.75
<u>Savings (<math>\% \Delta^{bench}</math>)</u>				
- Aggregate		-16.77	39.79	107.83
- Low		-19.68	43.09	116.85
- Medium		-18.35	43.90	119.05
- High		-12.25	30.95	83.65
<u>Output (<math>\% \Delta^{bench}</math>)</u>		-6.33	10.74	18.74
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## Concluding remarks

- Inequality in Australia is largely due to market income growth at the top.
  - Income gains at the bottom eaten away by bracket creep.
- Periods of accelerated growth and stagnation have impact on lifetime incomes.
  - Stable lifetime income inequality trend.
- Tax and transfer system reduces inequality but failed to completely curb its rise.
- Costs of income redistribution:
  - Disincentivizes the bottom and middle to work and save more.
    - Can result in higher market income inequality.

Need for more data and research on income dynamics

- **LINK TO OUR WEBSITE WITH DETAILED STATS**

**Thank You!**  
More Info @ Macro Public Finance Lab

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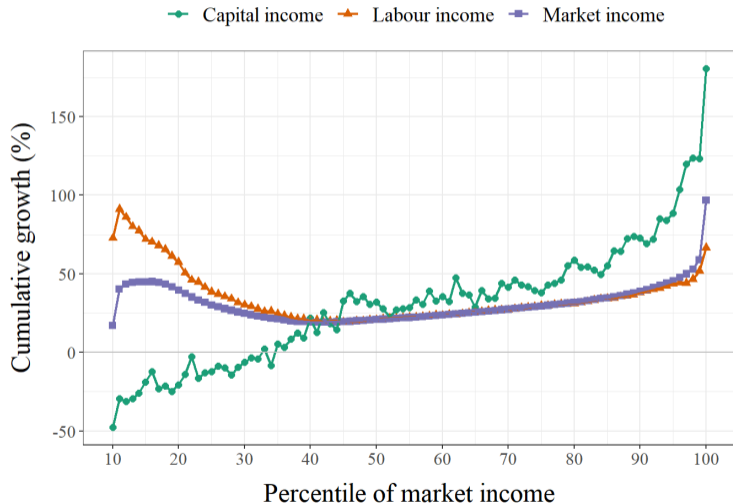
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# Appendix

# Cumulative growth in labour and capital income

Growth incidence curve 1991-2019







# Model details

## Demographics

- Age  $j \in [1, \dots, J]$ . In each period, a continuum of agents aged 1 are born and live upto a maximum of  $J$  periods.
- Constant population growth at rate  $n$ .
- Agents face survival probability  $\psi_j$  of surviving up to age  $j$  conditional on being alive at age  $j - 1$ .
- Fraction of population of age  $j$  at any point in time

$$\mu_j = \frac{\mu_{j-1}\psi_j}{(1+n)} \quad (8)$$

# Model

## PREFERENCES

$$U_0 = E \left\{ \sum_{j=1}^J [\beta^{j-1} \psi_j u(c_j, l_j) + (1 - \psi_j) \phi(b_{j+1})] \right\} \quad (9)$$

- Identical lifetime preferences over consumption  $c_j \geq 0$  and leisure  $l_j \in (0, 1]$ .
- Bequests are given by  $b(a_{j+1}) = a_{j+1}$  following De Nardi (2010)

$$\phi(b) = \phi_1 \left( 1 + \frac{b}{\phi_2} \right)^{1-\sigma} \quad (10)$$

- where  $\phi_1$  is the concern about leaving bequests,  $\phi_2$  measures the extent to which bequests are a luxury good.



# Model

## Endowments

- 3 skill types to match labor income quintiles

$$\rho \in \{low, medium, high\}$$

- Deterministic: Labor efficiency differs by skill type, and evolves over age

$$e_{\rho,j} : \text{age-dependent labor efficiency} \quad (11)$$

- Stochastic: shocks to labor efficiency within skill types

$$z_{\rho,j} = [low, medium, high]$$

$$\pi_j(z_{\rho,j+1}|z_{\rho,j})$$

- Effective labor services

$$h_j = (1 - l_j) e_j z_j \quad (12)$$

# Model

## Fiscal policy

1. Progressive income tax system (parametric tax function)

$$T(y_j) = y_j - \lambda y_j^{1-\tau} \quad (13)$$

2. Constant consumption tax rate  $\tau^c$ .
3. Means-tested pension
4. Public transfers to those below 65 years  $st_{\rho,j}$ : (exogenous, match public transfer shares by skill types and shocks)

# Model

## Means-tested pension

$$\mathcal{P}(a_j, y_j) = \begin{cases} \min \{ \mathcal{P}^a(a_j), \mathcal{P}^y(y_j) \} & \text{if } j \geq j^P \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

- Asset test

$$\mathcal{P}^a(a_j) = \begin{cases} p^{\max} & \text{if } a_j \leq \bar{a}_1 \\ p^{\max} - \omega_a(a_j - \bar{a}_1) & \text{if } \bar{a}_1 < a_j < \bar{a}_2 \\ 0 & \text{if } a_j \geq \bar{a}_2 \end{cases} \quad (15)$$

- Income test

$$\mathcal{P}^y(y) = \begin{cases} p^{\max} & \text{if } y_j \leq \bar{y}_1 \\ p^{\max} - \omega_y(y_j - \bar{y}_1) & \text{if } \bar{y}_1 < y_j < \bar{y}_2 \\ 0 & \text{if } y_j \geq \bar{y}_2 \end{cases} \quad (16)$$

# Model

## Government budget constraint

### 1. Balanced budget

$$\begin{aligned} \sum_j T(y_j) \mu(\chi_j) + \sum_j T(c_j) \mu(\chi_j) \\ = \sum_j \mathcal{P}(\chi_j) \mu(\chi_j) + \sum_j st_j \mu(\chi_j) + G + rD \end{aligned} \quad (17)$$

### 2. Written in terms of the scale of the income tax

$$\lambda = \frac{\sum_j y_j \mu(\chi_j) + \sum_j T(c_j) \mu(\chi_j) - \text{Expenses}}{\sum_j y_j^{(1-\tau)} \mu(\chi_j)} \quad (18)$$

# Model

## Firms and market structure

- Single representative firm

$$\max_{K,H} \{AF(K, H) - qK - wH\}$$

- One-period riskless asset: imperfectly self-insure against idiosyncratic earnings risk and mortality risks.
- Small open economy:
  - free flow of financial capital
  - domestic interest rate is equal to the world interest rate  $r$  such that rental price of capital is

$$q = r + \delta$$

## Household's problem

- Let  $\chi_j = (e_j, z_j, j)$  denote agent's state variables at age  $j$ .

$$V^j(\chi_j) = \max_{c_j, l_j, a_{j+1}} \{ u(c_j, l_j) + \beta \psi_j E[V^{j+1}(\chi_{j+1}) | e_j] + (1 - \psi_j) \phi b(a_{j+1}) \} \quad (19)$$

subject to

$$a_{j+1} = a_j + e_j(1 - l_j)w + ra_j + b_j + st_j + \mathcal{P}(a_j, y_j) - T(y_j) - (1 + \tau^c)c_j \quad (20)$$

$$a_j \geq 0, 0 < l_j \leq 1 \quad (21)$$

# Equilibrium

1.  $\{c_j(\chi_j), l_j(\chi_j), a_{j+1}(\chi_j)\}_{j=1}^J$  solve the household problem;
2. The firm chooses labor and capital inputs to solve the profit maximization problem;
3. Total lump-sum bequest transfer is equal to the total amount of assets left by all deceased agents  
Current account is balanced and foreign assets  $A_f$  freely adjust so that  $r = r^w$ , where  $r^w$  is the world interest rate;
4. Domestic market for capital and labor clear
5. The government budget constraint is satisfied

# Functional forms and calibration

## Summary

- Model is calibrated to match key features of the Australian economy 2000 - 2016.
- One model period equals 5 years. Agents enter model at age 20 and live a maximum up to 90 years. Eligible for pension at age 65.
- Survival probabilities from Life Tables 2003-2016 (ABS)
- Annual growth rate  $n = 1.56\%$  , long run average population growth (ABS)
- Labor efficiency and transition probabilities derived from hourly wage data (HILDA 2001-2016).
- Firms Cobb-Douglas production function

$$Y = AK^\alpha H^{1-\alpha}$$

- Fiscal parameters calibrated to match fiscal targets and income distribution (see benchmark model performance).



# Functional forms

## Preferences

- Instantaneous utility obtained from consumption and leisure

$$u(c_j, l_j) = \frac{\left[ (1 + d_j)^{\eta\gamma} c_j^\gamma l_j^{1-\gamma} \right]^{1-\sigma}}{1 - \sigma} \quad (22)$$

$\gamma$  - consumption weight,  $d_j$  - average dependent children by age,  $\eta$  is adjustment for children's consumption,  $\sigma$  - relative risk aversion.

- Utility from bequeathing

$$\phi(b) = \phi_1 \left( 1 + \frac{b}{\phi_2} \right)^{1-\sigma} \quad (23)$$

$\phi_1$  - concern over leaving bequests,  $\phi_2$  - extent to which bequest is a luxury good.