Progressive Pension and Optimal Tax Progressivity

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A progressive tax and transfer system

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- Little attention on progressive transfers
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 - Financed directly by progressive income tax.

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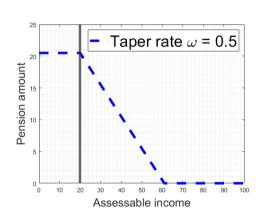
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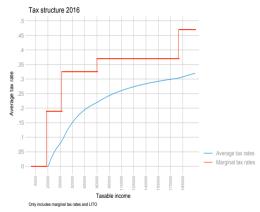
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Australia: the ideal labratory for this question





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2. General insights on effective tax-transfer design:

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- 2. General insights on effective tax-transfer design:
 - Address redistribution concerns directly via progressive (targeted) transfers.
 - Improve efficiency via reducing tax progressivity.
 - More revenue to fund transfers. (See my other paper on "Fiscal Limits").

Related literature

Optimal income tax

Varian (1980) Ventura (1999), Benabou (2002), Conesa and Krueger (2006), Krueger and Ludwig (2016), Heathcote and Tsujiyama (2022) and Heathcote, Storesletten and Violante (2017a)

Optimal pensions

Imrohoroglu, Imrohoroglu and Jones (1995), Sefton and van de Ven (2008), Kudrna and Woodland (2011), Tran and Woodland (2014)

Optimal tax and optimal transfers

McKay and Reis (2016) unemployment benefits, Jung and Tran (2022) social health insurance, Ferriere et al. (2022) means-tested transfers and income tax progressivity

Data and stylized facts

How progressive are income tax and pension in Australia?

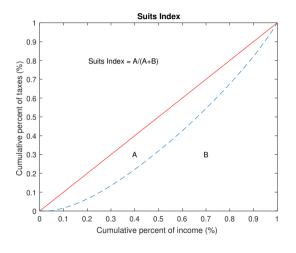
Data

- Administrative data from ALife (1991-2019).
- $\sim 0.8-1.1$ million obs. per year. (10% sample of all tax filers)

Method

- Suits index of tax progressivity
- Suits index of pension progressivity
- Parametric tax function

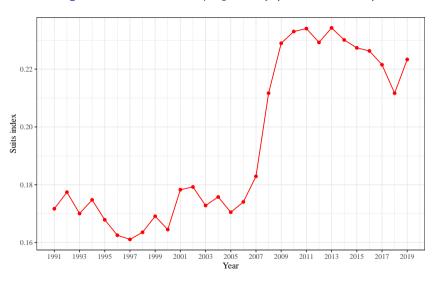
Measuring tax progressivity



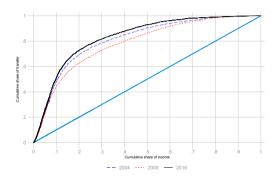
Suits index

- How are tax liabilities shared across income distribution?
- Suits = 0
 - Equally shared
 - Proportional
- Suits
 ightarrow 1
 - Concentrated at the top
 - More progressive

Figure: Suits index of tax progressivity (ALife 1991-2019)



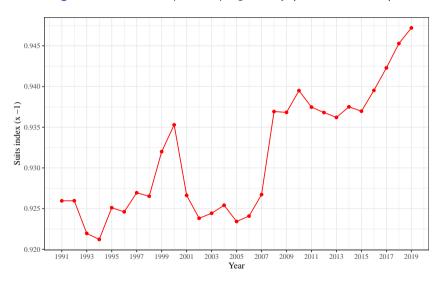
Measuring pension progressivity



Suits index

- How are pensions distributed?
- Suits = 0
 - Equally distributed
 - Universal coverage
- Suits $\rightarrow |1|$
 - Concentrated at the bottom
 - More progressive

Figure: Suits index of pension progressivity (ALife 1991-2019)



Progressivity of the tax code

- Different from distributional measures. (Often misunderstood).
- Progressive tax code (complex):
 - multiple thresholds, rising statutory marginal tax rates.
 - various offsets and credits for low incomes.

Progressivity of the tax code

- Different from distributional measures. (Often misunderstood).
- Progressive tax code (complex):
 - multiple thresholds, rising statutory marginal tax rates.
 - various offsets and credits for low incomes.
- Approximate using parametric tax function.
 - Jakobsson (1976), Persson (1983), Heathcote, Storesletten and Violante (2017b)

Parametric tax function

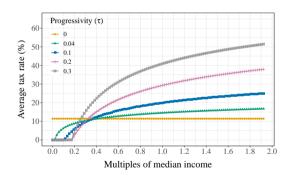
Total tax liability t(y) at income level y

$$t(y) = \max \left[0, y - \frac{\lambda}{\lambda} y^{(1-\tau^{y})}\right]$$
 (1)

$$atr = 1 - \frac{\lambda}{y} y^{-\tau^{y}}$$
 (2)

$$mtr = 1 - \frac{\lambda}{\lambda} (1 - \tau^{y}) y^{-\tau^{y}}$$
 (3)

Two parameters



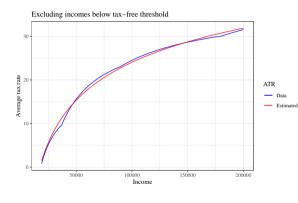
au^y controls progressivity

$$\begin{array}{c} \downarrow \tau^{y} \Longrightarrow \text{less progressive} \\ \Longrightarrow \downarrow \left(\lambda^{\frac{1}{\tau^{y}}}\right) \text{tax-free threshold} \end{array}$$

λ average level of taxation

$$1 - (\downarrow \lambda) y^{-\tau^y} = \uparrow atr$$

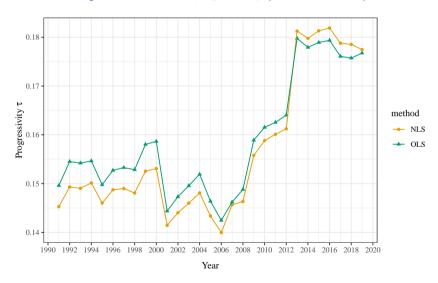
Good fit for Australian tax code



OLS estimates of τ^y

	Year	$ au^y$	95% CI	Adj. R ²
	1991	0.152	(0.151,0.152)	0.97
1	2000	0.150	(0.150,0.151)	0.98
	2010	0.129	(0.129, 0.129)	0.99
	2019	0.165	(0.165,0.166)	0.99

Figure: Trends in τ^y progressivity (ALife 1991-2019)



Progressive pension system

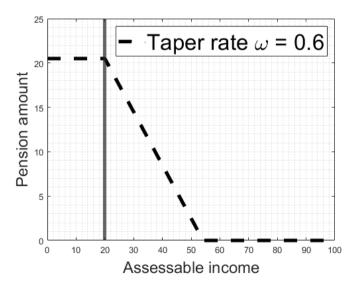
$$p = \begin{cases} p^{\max} & \text{if } y^m \leq \bar{y}_1 \\ p^{\max} - \omega^y (y^m - \bar{y}_1) & \text{if } \bar{y}_1 < y^m < \bar{y}_2 \\ 0 & \text{if } y^m \geq \bar{y}_2 \end{cases}$$
(4)

 p^{max} : maximum benefit. ω^y : taper rate.

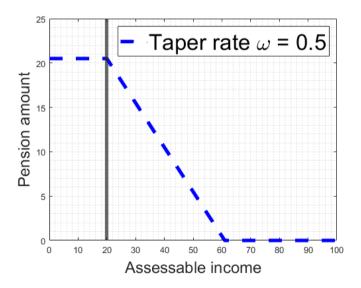
Eligible $j \ge 65$

(Income test is the binding one for majority).

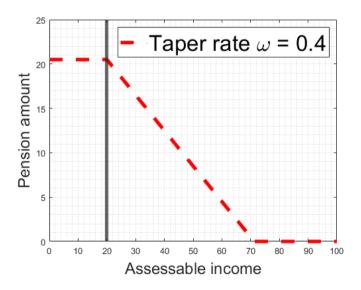
$\downarrow \omega^y$ makes pension less progressive



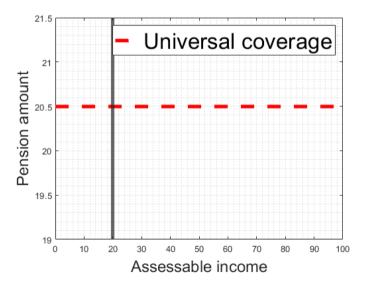
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$\omega^y = 0$



Tax and pension design in three parameters

Optimal combination of...

- Tax progressivity τ^y
- Pension progressivity ω^y
- Pension generosity p^{max}

Need a model

- Stochastic Overlapping Generations model for Australia (SOLGA)

SOLGA model

- Large scale computable general equilibrium OLG model (Auerbach and Kotlikoff, 1987)
- Heterogenous households who face uninsurable labour productivity risk. (Bewley, 1986; Huggett, 1993; Aiyagari, 1994)
- Representative firm
- Government (Australian tax-and-transfer system)
- Small open economy (foreign capital flows).
- Balanced growth path, steady-state equilibrium and stationary demographic structure.

Key model features and why we need them

Feature	Why we need them
Age $j \in [1,,J^p,,J]$	Need lifecycle (pension eligibility at age J^p)
Labor productivity risk Hump-shaped productivity	Social insurance role and incentives to work and save
Representative firm Foreign capital flows	General equilibrium effects Small open economy
Income tax, pension, transfers $<$ 65 yrs	Australian tax-transfer system

Labour productivity

Innate skill types

$$\underline{\varrho} \in \{\mathit{low}, \mathit{mid}, \mathit{high}\}$$

Labour productivity

Quintiles by age (hump-shaped)
$$\overbrace{\eta_{z,j} \in \{\eta_{1,j}, \eta_{2,j}, \eta_{3,j}, \eta_{4,j}, \eta_{5,j}\}}^{\text{Quintiles by age (hump-shaped)}} \underbrace{\pi_{z,j}^{\varrho} \left(\eta_{z,j+1} | \eta_{z,j}\right)}_{\text{Z}}$$

Transition probability matrix (differs by skill type)

Household choices

Saving, leisure, consumption over lifecycle

$$a_{j+1} = ra_j + \eta_{z,j} \left(1 - \frac{l_j}{l_j} \right) w + p_{j \ge J^p} + st_{j < J^p} - t \left(y_j \right) - \left(1 + \tau^c \right) c_j + a_j$$

 $a_j \ge 0, 0 < l_j \le 1$

Household incomes

$$a_{j+1} = \underbrace{ra_j + \eta_{z,j} \left(1 - l_j\right) w}_{y_j^m ext{(market income)}} + st_{j < J^p} + st_{j < J^p} - t \left(y_j\right) - \left(1 + au^c\right) c_j + a_j$$
 $a_j \ge 0, 0 < l_j \le 1$

Transfers to households

$$a_{j+1} = y_j^m + p_{j \ge J^p} + st_{j < J^p} - t(y_j) - (1 + \tau^c)c_j + a_j$$

- Public transfers before 65 years (progressive)

$$st_{j < J^p} = st(j, \eta_{z,j})$$

- Pension 65 and above

$$p = \begin{cases} p^{\max} & \text{if } y^m \leq \bar{y}_1 \\ p^{\max} - \omega^y (y^m - \bar{y}_1) & \text{if } \bar{y}_1 < y^m < \bar{y}_2 \\ 0 & \text{if } y^m \geq \bar{y}_2 \end{cases}$$

Taxes on households

$$a_{j+1} = y_j^m + p_{j \ge J^p} + st_{j < J^p} - t(y_j) - (1 + \tau^c) c_j + a_j$$

- Income tax

$$t(y_j) = \max \left[0, y_j - \lambda y_j^{1-\tau^y}\right]$$

- Consumption tax

Household problem

$$V^{j}(\chi_{j}) = \max_{c_{j}, l_{j}, a_{j+1}} \left\{ u(c_{j}, l_{j}) + \beta \psi_{j+1} \sum_{\eta_{z,j+1}} \pi_{z,j}^{\varrho} (\eta_{z,j+1} | \eta_{z,j}) V^{j+1}(\chi_{j+1}) \right\}$$
 subject to:

$$a_{j+1} = \underbrace{ra_j + \eta_{z,j} \left(1 - l_j\right) w}_{y_j^m ext{(market income)}} + st_{j < J^p} + st_{j < J^p} - t \left(y_j\right) - \left(1 + au^c\right) c_j + a_j$$
 $a_j \ge 0, 0 < l_j \le 1$

Government

$$Tax = \sum_{j}^{\text{Income tax} = \Sigma \left(\mathbf{y} - \lambda \mathbf{y}^{1-\tau^{y}} \right)} + \sum_{j}^{\text{Consumption tax}} + \underbrace{\tau^{f} \left(AK^{\alpha}H^{1-\alpha} - wH \right)}^{\text{Company income tax}}$$
(6)

$$Expenses = \sum_{j}^{\text{Age-pension}} p_{j} \left(y_{j}^{m} \right) \mu \left(\chi_{j} \right) + \sum_{j}^{\text{Other public transfers}} st_{j} \left(\eta_{j}, j \right) \mu \left(\chi_{j} \right) + \overbrace{G + rD}^{\text{Other expenses}}$$
(7)

Government budget balancing assumption

Adjust average level of income taxation $1-\lambda$ ("Revenue requirement")

$$(1 - \lambda) = 1 - \frac{\sum \mathbf{y} + \text{Consumption tax} + \text{Company tax} - \text{Expenses}}{\sum \mathbf{y}^{1 - \tau^{y}}}$$
(8)

Increasing pension expenditure \Longrightarrow Increasing $(1-\lambda)$

Rest of the model

- Standard.
- In appendix.

Key parameters

Parameter	Value	Source/Target
Preferences (Cobb-Douglas)		
Intertemporal elasticity of consumption	$\sigma = 2$	
Share parameter for leisure	$\gamma = 0.36$	Labour supply over the lifecycle
Discount factor	$\beta = 0.99$	Household savings share of GDP
Fiscal policy		
Consumption tax rate	$ au^c=10\%$	Consumption tax share of GDP
Income tax	$\lambda = 0.7237$	Income tax share of GDP,
	$ au^{y} = 0.2$	Suits index and Tax distribution
Company profits tax rate	$ au^f=11\%$	Company tax share of GDP and
		investment/GDP ratio.
Pension income test taper rate	$\omega^y=0.5$	Official taper rate
Maximum pension	p ^{max}	Pension share of GDP
Pension thresholds	, Y ₁	Pension participation rates
	, 1	• •

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Benchmark economy

Table: Key variables in the benchmark economy

Variable	Model	Targets
Investment	18.94	26.51
Consumption	54.87	56.30
Age-pension	2.62	2.54
Public transfers other than age-pension	6.49	6.42
Government debt	11.5	10
Personal income tax	11.4	11.4
Consumption tax	5.49	5.55
Company income tax	4.40	4.25
Suits index (Income tax distribution)	0.3	0.3
Gini coeficient (Taxable income)	0.44	0.45
Gini coeficient (Net income)	0.28	0.32

Note: All variables are expressed in terms of percentage of GDP. Data are averages of annual variables from 2012-2016.

Experiments

- Government policy Γ parametrized by $(\lambda, au^y, \omega^y, p^{max})$
- Welfare criterion: Ex-ante lifetime utility of agent in steady-state stationary equilibrium

$$SWF = \int V(\chi_{j=1}|\Gamma) d\Lambda(\chi_{j=1})$$
 (9)

- Compare consumption equivalent variation (CEV%) between counterfactual economies.
 - Aggregate (Utilitarian SW)
 - By skill type

Experiment 1

E1: Optimal tax progressivity with benchmark pension

- Vary $au^y \in [0,1]$

E2: Optimal tax and pension progressivity

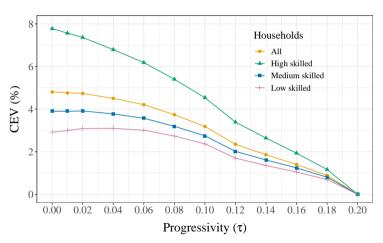
- Vary $au^y \in [0,1]$ and $\omega^y \in [0,1]$

E3: Optimal tax and pension progressivity and pension generosity

- Vary $(\tau^y, \omega^y, p^{max})$

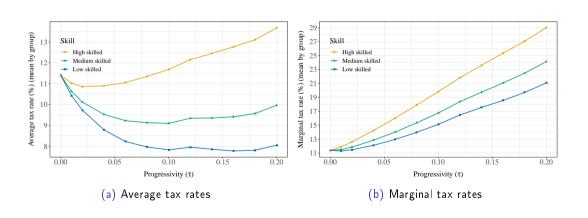
E1: Change in social welfare

Figure: Aggregate welfare gains and across skill types



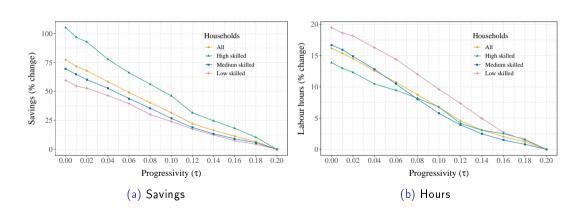
E1: Change in tax burdens and distortions

Figure: Average and marginal tax rates by skill type at different levels of au



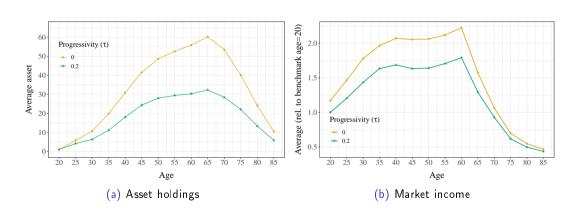
E1: Efficiency gains across skill types

Figure: Change in labour supply and savings as au changes



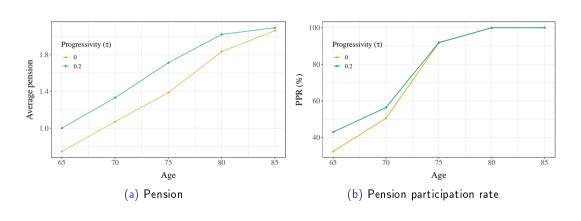
E1: Efficiency gains over lifecycle

Figure: Average asset holdings and market income by age (flat vs. benchmark)



E1: Less reliance on pension

Figure: Average pension and pension participation rate by age (flat vs. benchmark)



Experiment 2

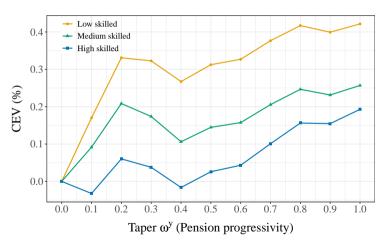
- E1: Optimal tax progressivity with benchmark pension
 - Vary $au^y \in [0,1]$

- E2: Optimal tax and pension progressivity (very briefly)
 - Vary $au^y \in [0,1]$ and $\omega^y \in [0,1]$

- E3: Optimal tax and pension progressivity and pension generosity
 - Vary $(\tau^y, \omega^y, p^{max})$

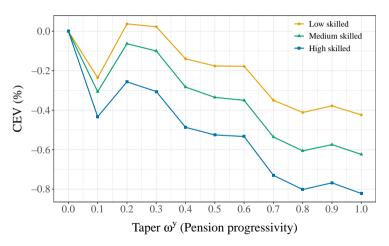
E2: Less progressive tax requres more progressive pensions

Figure: $\tau^y = 0$ (no social insurance role for income tax)



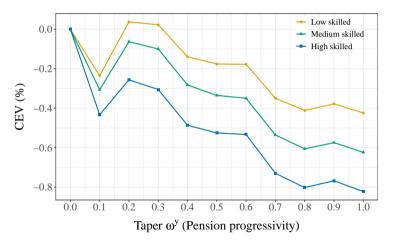
E2: More progressive tax requires less progressive pensions

Figure: $\tau^y = 0.3$ (When tax is more progressive than benchmark)



E2: More progressive tax requires less progressive pensions

Figure: $\tau^y = 0.3$ (When tax is more progressive than benchmark)



Experiment 3

- E1: Optimal tax progressivity with benchmark pension
 - Vary $au^y \in [0,1]$

- E2: Optimal tax and pension progressivity
 - Vary $au^y \in [0,1]$ and $\omega^y \in [0,1]$

- E3: Optimal tax and pension progressivity and pension generosity
 - Vary $(\tau^y, \omega^y, p^{max})$

E3: The optimal tax and pension design

Pension system

- Lower p^{max} (less generous)
- Higher ω^y (more progressive)
 - Lower pension expenditure

Income tax

- Lower cost o Lower average tax rates
- Less progressive
- BUT not completely flat
 - Less generous pensions creates need for social insurance from progressive income tax

E3: The optimal tax and pension design

	$\varphi^P = 0$	$\varphi^P = 0.5$	$\varphi^P = 1$	$\varphi^P = 1.5$
$ au^{y*}$	0.04	0.02	0	0
ω^{y*}	NA	0.9	1	0.2
Welfare (CEV%)				
Aggregate	5.26	5.56	4.94	4.10
Low skilled	2.09	2.96	3.03	2.82
Medium skilled	4.23	4.56	4.02	3.36
High skilled	9.39	9.24	7.96	6.32
Average tax rate % (mean)				
Aggregate	5.95	7.34		
Low skilled	5.09	6.80	10.92	14.76
Medium skilled	5.73	7.19	10.92	14.70
High skilled	6.87	7.92		

Robustness checks

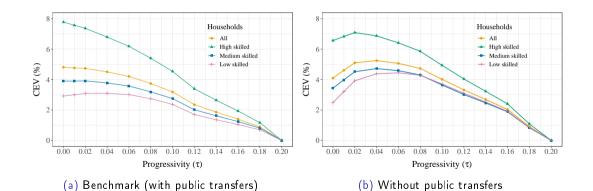
Labour supply elasticity

$$u(c, l) = \frac{\left[c^{\gamma} l^{1-\gamma}\right]^{1-\sigma}}{1-\sigma}$$
, Frisch elasticity is given by $\frac{l}{1-l} \frac{1-\gamma(1-\sigma)}{\sigma}$

	$\sigma = 1.5$	$\sigma=2$ (benchmark)
Elasticity for average HH	1.5	1.37
Tax progressivity $ au^{y*}$	0	0.02
Pension taper rate ω^{y*}	1	1
Pension level $arphi^p$	0.5	0.5
Welfare (%CEV)	6.32	5.56
Savings (% $ riangle$)	129	106
Hours (%△)	21	20

Robustness checks

Switch off public transfers



Concluding remarks

This paper

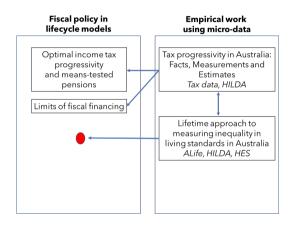
- Optimal design of income tax and means-tested pension

Main findings

- Tax should be less progressive ($\tau^{optimal} \in [0, 0.04]$).
- Lower progressivity ⇒
 - efficiency gains over lifecycle
 - less reliance on pension in old-age
- Address social insurance via progressive transfers
 - Strict means-tested pension

Lots of ongoing and planned extensions....

Thank you



I'm on the job market

- Website: www.nabeehz.com
- Email: nabeeh.zakariyya@anu.edu.au
- At the AEA meetings New Orleans

Appendices

- Intensive and extensive margin effects of means-tested pension
- Estimation of labour productivity
- Tax liability progression (more details)

Estimation of labour productivity

Nishiyama and Smetters (2007)

1. For each wave of the HILDA survey, we group individuals by skill type, age and quintile. Let $N_{j,s}^{i=\nu}$ be the total number of individuals of skill type s and age j in quintile $i=\nu\in[1,2,3,4,5]$.

$$\pi_{j,j+1}\left(e_{j+1}^{i=k}|e_{j}^{i=v}\right) = \frac{n_{j+1,s}^{i=k}}{N_{i,s}^{i=v}} \tag{10}$$

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- 2. Next, we track the movement of individuals in each group from age j to j+1. That is, we see whether they have stayed in one quintile or moved to another, and if so, which quintile they moved to. Let $n_{j+1,s}^{i=k}$ be the total number of individuals in the pool $N_{j,s}^{i=v}$ in age j that moved to quintile $i=k\in[1,2,3,4,5]$ at age j+1.

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- 3. The transition probability from quintile v at age j to quintile k at age j+1 is then calculated as

$$\pi_{j,j+1}\left(e_{j+1}^{i=k}|e_{j}^{i=\nu}\right) = \frac{n_{j+1,s}^{i=k}}{N_{j,s}^{i=\nu}} \tag{10}$$

Intensive and extensive margin effects of $\uparrow \omega^y$

Intensive margin

- Increases implicit tax rate ⇒ Lower labour & savings

Extensive margin

- Positive effect: Save/work more (because ineligble)
- Negative effect: Save/work less (to be eligible)

Interaction of $\downarrow \tau^y$ with ω^y

Intensive margin

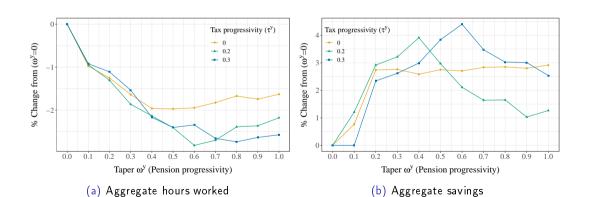
- Lowers implicit marginal tax rate ⇒ Increase labour & savings

Less reliance on pension (Extensive margin)

- Positive effect: Save/work more (because ineligble)
- Negative effect: Save/work less (to be eligible)

Interaction of $\downarrow \tau^y$ with ω^y

Figure: Savings and labour supply effects of increasing pension progressivity at different levels of tax progressivity



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The larger SOLGA Model Demographics

- Age $j \in [1, ..., J]$. In each period, a continuum of agents aged 1 are born and live upto a maximum of J periods.
- Constant population growth at rate n.
- Agents face survival probability ψ_j of surviving up to age j conditional on being alive at age j-1.
- Fraction of population of age j at any point in time

$$\mu_j = \frac{\mu_{j-1}\psi_j}{(1+n)} \tag{11}$$

$$U_{0} = E \left\{ \sum_{j=1}^{J} \left[\beta^{j-1} \psi_{j} u(c_{j}, l_{j}) + (1 - \psi_{j}) \phi(b_{j+1}) \right] \right\}$$
 (12)

- Identical lifetime preferences over consumption $c_i \geq 0$ and leisure $l_i \in (0,1]$.
- Bequests are given by $b(a_{j+1}) = a_{j+1}$ following De Nardi (2010)

$$\phi(b) = \phi_1 \left(1 + \frac{b}{\phi_2} \right)^{1 - \sigma} \tag{13}$$

- where ϕ_1 is the concern about leaving bequests, ϕ_2 measures the extent to which bequests are a luxury good.

Endowments

- 3 skill types to match labor income quintiles

$$\varrho \in \{\textit{low}, \textit{medium}, \textit{high}\}$$

- Deterministic: Labor efficiency differs by skill type, and evolves over age

$$e_{arrho,j}$$
 : age-dependent labor effiency

- Stochastic: shocks to labor efficiency within skill types

$$z_{arrho,j} = [ext{low, medium, high}] \ \pi_j\left(z_{arrho,j+1}|z_{arrho,j}
ight)$$

- Effective labor services

$$h_j = (1 - I_j) e_j z_j$$

(14)

(15)

Model Fiscal policy

1. Progressive income tax system (parametric tax function)

$$T(y_j) = y_j - \lambda y_j^{1-\tau} \tag{16}$$

- 2. Constant consumption tax rate τ^c .
- 3. Means-tested pension
- 4. Public transfers to those below 65 years $st_{\varrho,j}$: (exogenous, match public transfer shares by skill types and shocks)

Means-tested pension

$$\mathcal{P}\left(a_{j},y_{j}
ight)=egin{cases} \min\left\{\mathcal{P}^{a}\left(a_{j}
ight),\mathcal{P}^{y}\left(y_{j}
ight)
ight\} & ext{if } j\geq j^{P} \ 0 & ext{otherwise} \end{cases}$$

Asset test

$$\mathcal{P}^{a}\left(a_{j}
ight) = egin{cases} p^{\mathsf{max}} & \mathsf{if} \ a_{j} \leq ar{a}_{1} \ p^{\mathsf{max}} - \omega_{a}\left(a_{j} - ar{a}_{1}
ight) & \mathsf{if} \ ar{a}_{1} < a_{j} < ar{a}_{2} \ 0 & \mathsf{if} \ a_{i} > ar{a}_{2} \end{cases}$$

- Income test

$$\mathcal{P}^{y}\left(y
ight) = egin{cases} p^{\mathsf{max}} & ext{if } y_{j} \leq ar{y}_{1} \ p^{\mathsf{max}} - \omega_{y}\left(y_{j} - ar{y}_{1}
ight) & ext{if } ar{y}_{1} < y_{j} < ar{y}_{2} \ 0 & ext{if } y_{i} \geq ar{y}_{2} \end{cases}$$

(17)

(18)

(19)

Government budget constraint

1. Balanced budget

$$\sum_{j} T(y_{j}) \mu(\chi_{j}) + \sum_{j} T(c_{j}) \mu(\chi_{j})$$

$$= \sum_{j} P(\chi_{j}) \mu(\chi_{j}) + \sum_{j} \operatorname{st}_{j} \mu(\chi_{j}) + G + rD \qquad (20)$$

2. Written in terms of the scale of the income tax

$$\lambda = \frac{\sum_{j} y_{j} \mu\left(\chi_{j}\right) + \sum_{j} T\left(c_{j}\right) \mu\left(\chi_{j}\right) - \textit{Expenses}}{\sum_{j} y_{j}^{(1-\tau)} \mu\left(\chi_{j}\right)} \tag{21}$$

Firms and market structure

- Single representative firm

$$\max_{K,H} \left\{ AF\left(K,H\right) - qK - wH \right\}$$

- One-period riskless asset: imperfectly self-insure against idiosyncratic earnings risk and mortality risks.
- Small open economy:
 - free flow of financial capital
 - domestic interest rate is equal to the world interest rate *r* such that rental price of capital is

$$q = r + \delta$$

Household's problem

- Let $\chi_j = (e_j, z_j, j)$ denote agent's state variables at age j.

$$V^{j}(\chi_{j}) = \max_{c_{j},l_{j},a_{j+1}} \left\{ u(c_{j},l_{j}) + \beta \psi_{j} E\left[V^{j+1}(\chi_{j+1}) | e_{j}\right] + (1 - \psi_{j}) \phi b(a_{j+1}) \right\}$$

subject to

$$a_{j+1} = a_j + e_j (1 - l_j) w + ra_j + b_j + st_j + \mathcal{P}(a_j, y_j) - \mathcal{T}(y_j) - (1 + \tau^c) c_j$$

$$a_i > 0, 0 < l_i < 1$$
(24)

(22)

Equilibrium

- 1. $\{c_j(\chi_j), l_j(\chi_j), a_{j+1}(\chi_j)\}_{j=1}^J$ solve the household problem;
- 2. The firm chooses labor and capital inputs to solve the profit maximization problem;
- 3. Total lump-sum bequest transfer is equal to the total amount of assets left by all deceased agents

 Current account is balanced and foreign assets A_f freely adjust so taht $r = r^w$, where r^w is the world interest rate;
- 4. Domestic market for capital and labor clear
- 5. The government budget constraint is satisfied

Functional forms and calibration Summary

- Model is calibrated to match key features of the Australian economy 2000 2016.
- One model period equals 5 years. Agents enter model at age 20 and live a maximum up to 90 years. Eligible for pension at age 65.
- Survival probablities from Life Tables 2003-2016 (ABS)
- Annual growth rate n=1.56% , long run average population growth (ABS)
- Labor efficiency and transition probabilities derived from hourly wage data (HILDA 2001-2016).
- Firms Cobb-Douglas production function

$$Y = AK^{\alpha}H^{1-\alpha}$$

- Fiscal parameters calibrated to match fiscal targets and income distribution (see benchmark model performance).

Functional forms

Preferences

Instantenous utility obtained from consumption and leisure

$$u(c_j, l_j) = \frac{\left[(1 + d_j)^{\eta \gamma} c_j^{\gamma} l_j^{1 - \gamma} \right]^{1 - \sigma}}{1 - \sigma}$$
(25)

 γ - consumption weight, d_j - average depedent children by age, η is adjustment for children's consumption, σ - relative risk aversion.

- Utility from bequething

$$\phi(b) = \phi_1 \left(1 + \frac{b}{\phi_2} \right)^{1 - \sigma} \tag{26}$$

 ϕ_1 - concern over leaving bequests, ϕ_2 - extent to which bequest is a luxury good.